

## CALIBRATION OF AN ULTIMATE LIMIT STATE FOR MOORING LINES

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### ABSTRACT

Structural reliability analysis has been used to calibrate a design equation for mooring lines in their ultimate limit state. The calibration is based on six test cases, for mooring systems in water depths ranging from 70m to 2000m. Three of the cases apply to a turret-positioned ship and three to a semisubmersible. Conventional catenary mooring systems with chain and/or wire components have been studied, whereas taut moorings with fibre rope are not yet included. Environmental conditions from the Norwegian continental shelf and from the Gulf of Mexico have been considered. A design equation format involving two partial safety factors, applied to two tension components is recommended. The two components are: (i) the static tension due to pretension and due to tension induced at the offset position corresponding to the mean environmental forces in an environmental state, and (ii) the dynamic tension component due to time-varying loads; i.e. in this paper defined as the sum of time-varying low-frequency and wave-frequency tensions in the environmental state. The recipes for characteristic values of the tension components and the line capacity are specified, and partial safety factors are given.

### INTRODUCTION

Three criteria should be considered in the structural design of mooring lines for floating offshore structures. Within a structural reliability format it is convenient to formulate these criteria as: (a) an ultimate limit state (ULS) that to ensure that each mooring line is strong enough to withstand the extreme loads it is subjected to, (b) a progressive collapse limit state (PLS) to ensure that the mooring system can withstand the failure of one mooring line due to other causes, and (c) a fatigue limit state (FLS) to ensure that each mooring line has adequate capacity against fatigue. This paper deals with the ULS, while a companion paper deals with the PLS (Mathisen et al. 1998). The results are intended for use in the revision of the Posmoor rules for mooring line design (Sogstad 1998). There is also usually a serviceability requirement in the design of mooring lines, to ensure that the motion of the platform does not exceed limits imposed by attached risers or adjacent structures. This is obviously essential for a satisfactory design, but it is convenient to separate the serviceability requirement from the requirements placed on the strength of the mooring lines. The serviceability is usually adjusted by means of the line pretension, elasticity, weight, or number of lines. After changing any of these

parameters it is necessary to check that all limit states are still satisfied.

The objective of this work is to calibrate a simplified design method for the ULS, against a detailed structural reliability analysis of the ULS, such that a chosen target reliability level is achieved when the design method is applied. In order to ensure applicability of the rules in very deep water, the test set includes water depths down to 2000 m. The reliability analysis and calibration of the ULS is based on the experience obtained from two preceding joint industry projects: (a) FPS 2000 project 1.8, Reliability of Station Keeping, Systems, Mathisen (1992), and (b) PROMOOR, Reliability-Based Design of Mooring Systems, Mathisen and Hørte (1996). The present paper therefore focuses on the calibration results.

### CALIBRATION

Madsen et al. (1986) provide an introduction to calibration. Discussion of calibration can be a little confusing, because two methods of analysing the same problem are involved. The essential difference is that one method is simplified to make it convenient in practical design, while the other method is detailed to carry out the analysis in the best way available within the state-of-the-art. The calibration typically involves adjusting the partial safety factors applied in the design method, so that the resulting designs are close to a chosen target reliability level. The calibration can also be generalised to include other adjustments to the design method, such as changes in the format of the design equation, or in the definitions of the characteristic values that are involved. The calibration process may be considered to be a mathematical optimisation process, to minimise an objective function measuring the distance of the resulting designs away from the target reliability.

It is advisable to clearly define the scope of the calibration; i.e. the class of structures that the design method is intended to be applicable to. Here, we intend to encompass mooring systems for floating offshore structures, in water depths from 70 m to 2000 m, using conventional chain and steel wire rope mooring line components. Semisubmersibles and ships are included, while tension leg platforms are excluded. We had hoped to include Spar platforms too, but have not had the resources to include a Spar platform in the test set yet. The results are also intended to be applicable world wide, provided that the recipe for characteristic values including the environmental conditions is followed. Strictly speaking, the calibration should be checked if the design

rule is to be applied in locations where the distribution of environmental conditions falls outside the range covered by the two cases that have been used.

A set of test structures have been selected to span the scope of the calibration. These include a turret-positioned ship and a semisubmersible, with various mooring systems for water depths of 70 m, 350 m, 1000 m and 2000 m. Environmental conditions for the Norwegian continental shelf and the Gulf of Mexico are included.

The calibration is carried out as an iterative process, as indicated in Fig.1. In practice, there is a need to simplify the iteration process, to avoid excessive computations in each iteration loop. This has been done by assuming that the mooring system response is unaffected by perturbations in the mooring line strength. The reliability analysis can then be carried out beforehand for a few line strengths, and the reliability results needed in the iteration loop can be provided by interpolation. The same assumption is also made in the design analysis, for consistency.

## RELIABILITY ANALYSIS

### Limit State Model

The present application is concerned with the probability of failure of a single mooring line, under extreme environmental conditions, without prior failure of any of the other mooring lines. This failure mode is referred to as an ultimate limit state (ULS). The ULS is intended to give a sound design of each individual mooring line to withstand the extreme loads it is expected to be exposed to. Mooring system failure is not considered in the ULS, but will be covered by the PLS. The PLS takes account of the possibility that a mooring line may fail due to some exceptional or unknown cause not accounted for in the ULS, and ensures that the mooring system has an ability to withstand such incidents. (Note that the empirical probability of failure of one line is relatively high and needs to be taken into account in the serviceability limit state. This can be done conservatively by assuming that any single line may be missing.) The ULS formulation used was originally developed by Braathen and Mathisen (1991), as has been presented by Mathisen and Mørk (1993), and Larsen and Mathisen (1996b). Some additional details of the formulation are included in the description by Mathisen and Hørte (1994).

Tensile overload failure in any component of the line is included, and the distribution function for the strength of each type of component is required as input. The strengths of individual components are assumed independent. Variation in tension along the length of the line is neglected, based on the results of previous analyses, where it was taken into account, Mathisen and Hørte (1994).

The probability of failure is calculated by integration over the joint probability distribution of the environmental effects (long term type analysis). Fig. 2 provides an overview of the ULS computation.

Only chain link and steel wire rope components from the main body of the line are included, because the design rule is to be calibrated for these types of components for predictable normal conditions. Connecting links and end terminations are omitted in the analysis. Mechanical wear and special load effects at fairlead as well as corrosion are not taken into account. Abnormal conditions are to be covered by PLS.

### Response Analysis

The main excitation sources for the mooring line are the vessel motions in the mooring line plane at the upper terminal point (fairlead). For a given state of the environmental variables, the low-frequency (LF) motion is the result of vessel low-frequency

surge, sway and yaw, while the wave-frequency (WF) motion is defined as the tangential motion transformed to the fairlead point.

The tension caused by the pretension, offset due to mean environmental forces and the low-frequency motion may be calculated by quasi-static analysis from the actual line characteristics. The tension due to the tangential WF motions at the fairlead is computed by a dynamic analysis. The total tension at the upper terminal point is the sum of the quasi-static and dynamically calculated tension.

Since both the quasi-static top tension due to low-frequency motion and the WF top tension are stochastic processes, some difficulty arises in making this combination. An approach is adopted, based on Turkstra's hypothesis, which states that the extreme value of the combination is expected to occur when the extreme value of one of the components occurs. Two combinations are considered:

CASE A: An extreme value of the quasi-static tension together with a local maximum of the WF tension

CASE B: An extreme value of the WF tension together with a random value from the parent distribution of the quasi-static tension

Note also that the WF tension is always conditional on the quasi-static tension. The combination is carried out in a short term, stationary environmental state.

TABLE 1 OVERVIEW OF STATISTICAL DISTRIBUTIONS.

Variable	Description	Distribution
$X_{wf}$	Tangential WF motion of the upper terminal point.	Parent: Gaussian
$X_{lf}$	Resulting LF motion of the upper terminal point.	Parent: Gaussian Extremes: Acc. to Stansberg (1991,1996)
$Z_D$	Dynamic tension due to tangential WF motions, $X_{wf}$ .	Parent: Gaussian Maxima: Rayleigh Extremes: Gumbel
$Z_S$	Quasi-static tension due to pretension, mean offset and LF motion, $X_{lf}$ .	Given by distribution for $X_{lf}$ and the line characteristics $z(x)$ .

The statistical distributions applied for the different response variables are summarized in Table 1. The short term variability of the extreme tension within a sea state is in this way properly accounted for. Model tests have shown that a realistic modeling of the low-frequency extremes may be somewhere between a Rayleigh and an Exponential distribution. This has been taken into account in the reliability analysis, and an additional correction of the offset due to non-linearity in the mooring system stiffness has also been implemented.

The mooring system response computations are performed with the MIMOSA program in the frequency domain, Marintek (1995). Environmental conditions for Haltenbanken and the Gulf of Mexico have been applied. Two different wave spectrum formulations were used for Haltenbanken; a single peaked formulation given by JONSWAP and a two-peaked spectral formulation as described by Torsethaugen (1996). The JONSWAP spectrum was used for the Gulf of Mexico. The 1-hour mean wind has been used together with a wind-spectrum (NPD). The surface current velocity has been applied.

The reliability analysis is carried out by means of the PROBAN program (DNV, 1996). The system response information is transferred between the system response analysis, carried out by MIMOSA, and the reliability analysis through interface files. The interface files carry the short term system

response for a set of discrete environmental states. A response surface module, Mathisen (1993), is applied to interpolate between these results, to provide the system response in any environmental state, as required in the reliability analysis. Fig. 2 shows an overview of the data flow through the computations. The system response information is stored on three interface files:

- 1) Mean position and LF response interface file (POSLF). Gives information about mean position and LF motion of the vessel as function of the environmental parameters.
- 2) Interface file for line characteristics (LCHAR). Gives the quasi-static tension as function of offset.
- 3) Dynamically calculated WF tension interface file (DYNRSP). Provides data necessary in order to estimate dynamic tension due to the WF motions of the vessel.

All relevant environmental conditions are to be covered on the interface files in order to make efficient iterations for the failure probability calculations. The accuracy of the response surfaces, at the design points (most likely combinations of variables at failure) for the individual cases from the reliability analysis, have been verified.

### Probabilistic Model

The limit state function for a single component number  $i$  of a mooring line can simply be expressed as the difference between the component strength  $S_i$  and the applied tension  $Z_i$

$$g_i(s_i, z_i) = s_i - z_i, \quad i = 1, 2, \dots, n \quad (1)$$

However, the very large number of components  $n$  usually present in a mooring line forms an important aspect of this reliability problem, which is best formulated directly in terms of the strength and tension variables, rather than indirectly from the component limit states.

The probability of the ULS mode of line failure  $P_f$  is given by the probability of failure in any component of the line. A component may be a chain link or a component of wire rope, with length typically 30 times the diameter. It is assumed that the same tension  $Z$  is applied to all the components in one line. Since the strengths of the components are assumed to be independent, and by conditioning on the top tension  $Z$ , then the component events are independent and the multiplication rule may be applied.

On this basis the conditional probability of failure is given by

$$P_{f|Z}(z) = 1 - \prod_{i=1}^{i=n} [1 - F_{S_i}(z)] \quad (2)$$

To obtain the marginal probability of failure (i.e. not conditional on the tension  $Z$ ), it is first necessary to obtain the distribution of the conditioning variable  $F_Z(z)$ . This is handled via the combined tension  $Z$  at the upper terminal point, taken as the sum of the quasi-static tension  $Z_S$  and the dynamic tension  $Z_D$  as specified by Turkstra's hypothesis.

The marginal probability of failure is then obtained from equation (3) through the theorem of total probability as

$$P_f = \int P_{f|Z,R}(z, r) f_{Z|R}(z|r) f_R(r) dz dr \quad (3)$$

where  $R$  is a vector of time-independent random variables, including all the model uncertainties that are applied, with joint probability density function  $f_R(r)$ , and  $f_{Z|R}(z|r)$  is the probability density of the annual extreme top tension - which is here shown conditional on the time-independent random variables  $R$ , because the detailed model uncertainties have to be applied before the combined tension is obtained. The annual extreme value distribution applies to the tension  $Z$ . Thus the result is an annual probability of failure.

## Uncertainty Modelling

### Capacity

The assumptions used for the capacity model are based on experience gained during previous studies, which has been reported more in detail within the DEEPMOOR project and its preceding projects. The assumptions are summarised in Table 2, where MBS refers to the minimum breaking strength.

Before the reliability analysis of line failure was carried out, the strength distribution of an entire line was computed using the above parameters taking into account the number of components in the line. This is permissible since tension variation along the line has been ignored. The strength distribution of the entire line is a rather narrow distribution, and is approximated by a normal distribution function, effectively expressed by equation (2).

TABLE 2 STRENGTH DISTRIBUTIONS OF MOORING LINE COMPONENTS

Type of component	Distribution type	Mean value	C. o. V.
Chain link (*)	Lognormal	1.2 MBS	5%
Steel wire (30 · D)	Lognormal	1.16 MBS	5%

(\*) A lower threshold equal to the proof load has been assumed.

### Loads and environmental conditions

The distribution of the line tension is computed from the distributions of the underlying vessel motions, as described previously. The tension distribution is initially conditional on the environmental variables. The joint distribution of the environmental variables, including wave, wind and current, is based on the model developed by Bitner-Gregersen and Haver (1989) and Bitner-Gregersen (1993). The wave heading has been assumed uniformly distributed around the circle. The model allows for different headings for wave, wind and current. Data for Haltenbanken and the Gulf of Mexico have been applied. The data for the Gulf of Mexico includes 1-hour sea states with significant wave heights above 8 m.

### Model uncertainty

Four model uncertainty variables are included in the reliability analysis, and are listed in Table 3. They are all applied as multiplicative factors to the respective system response variables, in the reliability analysis. The applied values are based on very well tuned models, where results from time domain analyses and model tests are used. Uncertainty in the pretension is assumed insignificant, and is not included in the analysis.

TABLE 3 DISTRIBUTIONS OF MODEL UNCERTAINTIES.

Variable name	Symbol	Distribution Type	Parameter Name	Param. Value
Factor on mean offset	U1	Normal	Mean CoV	1.0 0.05
Factor on std.dev. LF motion	U2	normal	Mean CoV	1.0 0.10
Factor on extreme LF motion	U3	normal	Mean CoV	1.0 0.10
Factor on dynamic WF tension	U4	Normal	Mean CoV (*)	1.0 0.10

(\*) At water depths of 1000 m or more a CoV of 0.05 was used because a more detailed analysis in terms of motion to tension transfer functions from a finite element analysis was used.

### Designer uncertainty

Some allowance should be made for the different system response results likely to be obtained by different designers following the recipes given for characteristic loads in the design rule. This is covered by the designer uncertainty factor. An estimate of the effect of designer uncertainty has been made by introducing a normal variable with a coefficient of variation of 10% and no bias. This variable was multiplied to the strength of the mooring lines. The impact of this variable was checked for a couple of the most critical cases for the calibration only. The ratio between the probability of failure with and without designer uncertainty was found to be around 1.6 at target reliability level. Although this ratio may vary somewhat from case to case, it is considered fairly representative for all cases. The effect of the designer uncertainty on the calibrated safety factors was found to be relatively small (less than 5% increase in the safety factors) because the designer uncertainty is combined with other, larger uncertainties in the reliability analysis.

The chosen level of designer uncertainty may be somewhat optimistic. This should be reflected by detailed recipes for characteristic values in the design rule, to limit the difference in characteristic values between different designers, and to encourage conservative input parameters when accurate data is unavailable.

### Reliability Results Required for Calibration

The main result from the reliability analysis is the probability of failure and the corresponding design point values of the random variables (most likely combination of random variables at failure). Analyses have been carried out for several values of the mooring line strength. A second order reliability method (SORM) has been used. A first order method (FORM) was not sufficiently accurate since the effect of random wave heading introduce a non-linear failure surface.

With the probability of failure available for various line capacities around the target reliability level, the calibration of safety factors is performed efficiently in a spreadsheet by interpolating on these results.

### DESIGN ANALYSIS

The design analysis is essentially based on the current design practice. However, several alternatives have been tested, and some modifications are proposed. The design analysis represents a design format that specifies:

- the algebraic form of the design equation and variables involved
- the recipe for the characteristic resistance and load-effect
- the values of the partial safety factors

### Design Equation

The code formats presented in this chapter are a single safety factor format as generally used in mooring codes today, and a format with two separate safety factors.

#### Single safety factor (format 1)

The simplest possible format for the mooring line overload problem just takes the difference of the characteristic line strength  $s_c$  and the characteristic line tension  $z_c$ , with corresponding safety factors  $\gamma_s$  and  $\gamma_z$

$$s_c / \gamma_s - z_c \cdot \gamma_z \leq 0 \quad (4)$$

Based on previous work,  $\gamma_s$  may be set to unity when the characteristic line strength is taken as the median value of the strength of the whole line. Equation (4) then involves only a single safety factor to be applied to the total tension.

#### Two safety factors (format 2)

This format is a refinement of the previous one, where the total characteristic tension is divided into two tension components. The first component is the line tension at the mean offset (offset due to mean environmental forces)  $z_{CM}$ , and the second is the increase in tension due to low-frequency and wave-frequency dynamic motions of the vessel  $z_{CD}$ . This format proposal is a result of the calibration work performed within DEEPMOOR. Assuming no safety factor on the characteristic line strength, format 2 may be written as:

$$s_c - z_{CM} \cdot \gamma_{ZM} - z_{CD} \cdot \gamma_{ZD} \geq 0 \quad (5)$$

where  $\gamma_{ZM}$  and  $\gamma_{ZD}$  are the safety factors for the mean and dynamic tension components respectively. Note that the "dynamic tension" has a different meaning here and in current usage, where it usually refers to the tension component excited by wave-frequency motions only.

### Characteristic Values

#### Characteristic line strength

TABLE 4 CHARACTERISTIC STRENGTH FOR TEST SETS USING THE PROPOSED EQUATION (6), NORMALISED TO THE MINIMUM BREAKING STRENGTH.

Type of component	Characteristic strength (normalised) based on equation (6)
Chain	1.03
Chain	0.96
Chain	0.84
Chain	1.06
Chain	1.06
Chain	1.00
Chain	0.97
Chain	0.90
Chain	1.06
Wire	0.99
Wire	1.03
Wire	0.94
Wire	1.15

The minimum breaking strength is currently applied as the characteristic strength in mooring line codes. This parameter only makes limited provision for differing quality of mooring line components; e.g. it varies with the grade of the chain link, but not within the same grade chain from different manufacturers. The reliability analyses demonstrate that consistent component quality, as quantified by the mean value and coefficient of variation of the breaking strength greatly affects the line reliability. It would also seem likely that both component manufacturers and designers would like to be able to take better account of the benefit of high quality components. Accordingly, a new definition of characteristic strength is suggested as

$$s_c = \mu_s \cdot [1 - \delta_s \cdot (3 - 6 \cdot \delta_s)] \quad \delta_s < 0.25 \quad (6)$$

where  $\mu_s$  is the mean strength and  $\delta_s$  is the coefficient of variation of the component strength. Fig. 3 gives a comparison of this expression with the median line strength and component minimum breaking strength. It may be seen that the median value of the distribution for the entire line is very sensitive to the coefficient of variation for the individual components. It also appears that the median value is not so sensitive to the line length, provided that a realistic minimum length of line is considered. Thus, it seems not essential to include the detailed effect of line length in the characteristic strength. The strength distribution of

the entire line is a rather narrow distribution, typically with a coefficient of variation below 2% if the coefficient of variation for individual line components (chain link of wire component) is 5%. The reliability analyses have shown that the design point (most likely value at failure) for line strength is close to the median strength of the entire line.

This suggested type of definition could also help to assess the suitability of old or used mooring line components through testing, for which the minimum breaking strength of new components is no longer known to be representative as a characteristic strength.

Limited breaking load test data are available. Some data have been collected and analysed. Based on statistics in terms of mean values and coefficients of variation for the strength of individual chain links and wire components, the characteristic line strength based on equation (6) has been calculated. This exercise is carried out in order to evaluate the use of equation (6) compared to the conventional specification of characteristic strength taken as the minimum breaking strength. The results are normalised to the minimum breaking strength, and included in Table 4. It is worth mentioning that the information behind the reported numbers in some cases is rather limited, and one should therefore not emphasise individual numbers too much. The data includes both new and old, used chain, e.g. the ratio of 0.84 is for used chain.

Equation (6) is recommended as a specification of characteristic strength provided that relevant test data are available for the component considered. If such data are unavailable, the characteristic strength should be set equal to the minimum breaking strength multiplied by a reduction factor of 0.95. Other factors may be needed for old lines.

### Characteristic tension

A preferable and consistent definition of the characteristic tension would be a certain fractile of the annual extreme value distribution of the tension. However, there is no straight forward method to compute this distribution. A procedure in accordance with present practice is therefore applied, in which short term environmental conditions with a certain return period are applied, and the characteristic tension is computed according to a predefined recipe.

In the current work a procedure based on inverse FORM (Winterstein et al. 1994) has been applied to establish a 100-year contour for significant wave height and peak wave period. This procedure requires that a joint environmental model is available for the significant wave height and peak wave period. The most critical point on this contour has been checked in combination with a 100-year wind and a 10-year current from their respective marginal distributions.

The computation of characteristic tension for these environmental conditions is based on the procedure given in the API-RP 2SK rules. Gaussian processes are here assumed both for the low-frequency motion and the dynamically calculated tension. This use of Gaussian processes is a considerable simplification which will, in some cases, lead to significantly lower line tension than provided by more refined methods. This distribution model is chosen because it is widely used, it is simple, and general consensus has not yet been achieved about a more refined model. The calibration process is intended to make allowance for this simplification in the computed partial safety factors - provided that the test set includes cases where the recipe leads to low tension.

Both dynamic analysis including line dynamics for the wave-frequency tension and quasi-static analyses have been performed. This paper emphasises the results from the dynamic analysis since quasi-static analysis is known to be inaccurate in deep water.

For the weather vaning ship, both collinear and non-collinear

environmental conditions have been checked. The non-collinear case is defined by wind at 30° and current at 45° relative to the wave direction. This is similar to the present specification in the POSMOOR rules.

### TEST CASES

Six different application examples have been selected to study. A ship of the storage tanker type and a typical semisubmersible, each moored at three different water depths, i.e. 70 m, 350 m and 2000 m for the ship and 70 m, 350 m and 1000 m for the semi. The main particulars for the ship and the semi are given in Table 5.

The ship is equipped with 8 mooring lines evenly spread around the circle. The semi has 12 mooring lines in groups of 3 at each corner, where the angle between the lines in a corner is 5 degrees. Separate mooring systems were specified for the ship and the semisubmersible at the different water depths. Realistic combinations of wire and chain segments were used, and the initial safety factors for the systems were typically chosen between 1.5 and 2.0 using present rules. A buoy was included for the case in 2000 m water depth.

TABLE 5 MAIN PARTICULARS OF THE SHIP AND SEMISUBMERSIBLE

ITEM	Ship	Semisubmersible
Length p.p. (m)	215	-
Length o.a. (m)	-	121
Breadth o.a. (m)	42	95.3
Draught (m)	16	21
Distance midship-turret (m)	90	-
Displacement (tonnes)	120000	52500
Number of Columns	-	4
Column cross-section (m)	-	16.6x16.6
Number of pontoons	-	4
Pontoon BxH (m)	-	16.6x8.4

### CALIBRATION RESULTS

#### Objective Function

The calibration has been performed using an objective function providing a high penalty for under-designed cases. It allows no cases to have a reliability level far below the target level. The objective function  $\Delta$  is given by:

$$\Delta(P_T, \bar{\gamma}) = \sum_{i=1}^N [P_T - P_i(\bar{\gamma})]^2 \quad (9.1)$$

where

$P_T$  is the target annual probability of failure

$\bar{\gamma}$  is the safety factor(s) subjected to calibration

$P_i$  is the annual probability of failure for case  $i$  associated with the calculated safety factor

$N$  is the number of cases included in the calibration analysis

#### Target Level

The basic objective of this ULS is to ensure that an ordinary mooring line is designed strong enough to withstand the tensions it is expected to be exposed to. The question of what happens under "exceptional" circumstances is not in the domain of the ULS, but is covered by PLS. The PLS is also intended to ensure that there is a reasonable amount of redundancy in the mooring system, if one mooring line should fail. In this context, the annual target probability of failure  $P_{fl} = 10^{-4}$  is chosen for the calibration of the ULS, with the corresponding target reliability index  $\beta_T = 3.71$ . This choice is based on the following:

- The reliability levels computed for designs at the minimum

safety factor required by current design codes.

- Experience obtained with the present mooring line reliability analysis through the preceding projects FPS 2000 and PROMOOR joint industry projects. Results from these projects have been published (Mathisen and Mørk 1993, Larsen and Mathisen, 1996a and 1996b) without attracting significant criticism. This mooring line reliability analysis also corresponds well with the models applied in analyses of other types of offshore structures, as recommended by Skjong et al. (1996).
- The values of acceptable annual probabilities of failure quoted in DNV Classification Note no. 30.6. The chosen category of target level in the Classification Note applies to a redundant structure, with serious consequences of failure.

### Characteristic and Design Point Tension

The reliability analyses show that the probability of failure is dominated by the variability in the load. Hence, the effectiveness of the design format will be dependent on how well the characteristic load takes account of the change in load variability from case to case. The characteristic tension has been calculated for the environmental conditions as listed in Table 6. Conditions "H" indicate Haltenbanken environment and "G" the Gulf of Mexico. Both collinear and non-collinear environmental conditions have been checked.

TABLE 6  
SELECTED 100-YEAR  
ENVIRONMENTAL CONDITIONS IN DESIGN CHECK

Condition no.	Waves (100-year contour)		Wind (100-year marginal) (m/s)	Current (10-year marginal) (m/s)
	H <sub>s</sub> (m)	T <sub>p</sub> (s)		
H1	16.5	20.8	39	0.51
H2	16.0	19.4	39	0.51
H3	13.0	14.9	39	0.51
H4	10.5	11.6	39	0.51
H5	9.8	10.7	39	0.51
H6	9.0	9.7	39	0.51
G1	12.95	14.83	44.12	2.57
G2	11.48	12.93	44.12	2.57
G3	9.98	11.52	44.12	2.57
G4	8.96	10.73	44.12	2.57

As a first exercise in terms of calibration, it may be helpful to compare the characteristic tension (dynamic analysis) with the corresponding design point tension, i.e. the most likely tension at the target probability of failure. This is done in Fig. 4. The two first letters indicate ship or semisubmersible by "sh" or "se" respectively. The number indicate water depth, and "ns" represents North Sea (Haltenbanken) cases, "gom" the Gulf of Mexico, "2p" for double peaked spectrum, "jo" for Jonswap and "hc" indicate that heading control has been applied. The ratio between design point and the characteristic tension indicate the required magnitude of a safety factor to achieve the target reliability. Unfortunately, it may be seen in Fig. 4 that this ratio is not the same for the various cases. The cases in shallow water seem generally to need a greater safety factor than the cases in deep water. In fact, some of the cases in deep water will achieve

the target reliability level even with a safety factor of unity. This may seem strange since the characteristic tension is defined in terms of a sort of fractile of the environmental distribution (100-year return period), and a combination of fractiles of the distributions of the tension components. However, the resulting characteristic tension does not turn out to be a consistent fractile of the annual extreme value distribution of the tension for the various cases. The difference arises because the change in mooring system response with water depth amplifies the various simplifications in the characteristic tension recipe in different ways.

Now consider the design equation format with the characteristic tension divided into two components. Fig. 5 compares characteristic and design point tension at mean offset. The design point tension at mean offset is usually equal to or lower than the characteristic tension. There appears to be relatively good correspondence in the results for most cases. A safety factor near unity for this tension component seems appropriate. For the cases where the largest discrepancies occur, the tension at mean offset is a rather small portion of the total tension. For the case of sh350-ns-jo the characteristic mean tension greatly exceeds the design point value. This arises due to a relatively large difference in the characteristic and the design point wind velocity. The design point wind velocity is 21.3 m/s, together with a significant wave height of 8.9 m and a peak wave period of 11.4 s. The critical environmental condition in the design check for this case is condition H5 in Table 6, where the marginal 100-year wind velocity of 39 m/s has been applied irrespective of the value of the significant wave height. A correspondingly higher value of the mean tension is therefore obtained. This particular combination of sea state and wind velocity in the design check is not realistic, but, is specified for simplicity, to avoid the need for joint distributions of wave and wind that may be unavailable.

Fig. 6 shows a comparison of characteristic and design point dynamic (low-frequency plus wave-frequency) tension. The design point tension is generally higher than the characteristic tension. A safety factor for this tension component needs to be well above unity for most of the cases.

Figs. 4 to 6 indicate that the design format with two safety factors will improve the consistency of the design rule with water depth compared to a single safety factor format. In deep water, a relatively large portion of the tension is pretension and tension due to mean environmental forces. These tension components are likely to be associated with relatively low uncertainty and need a lower safety factor than the dynamic low-frequency and wave-frequency tension.

### Safety Factors

#### Single safety factor

Design format 1 with a single safety factor  $\gamma_2$  applied to the characteristic tension is widely used in mooring design. Safety factors for this format have been calibrated based on quasi-static and dynamic analyses. The cases at 1000 m and 2000 m water depths are omitted in the calibration of safety factors for quasi static analysis

The safety factors are summarised in Table 7. Separate calibration results are reported for all cases included and for the Haltenbanken and Gulf of Mexico cases alone. The Gulf of Mexico requires a quasi-static safety factor of 1.67, whereas 1.60 is required at Haltenbanken. The factor is 1.66 when all cases are considered. In the case of dynamic analysis, the difference between the calibrated safety factors for Haltenbanken and the Gulf of Mexico is negligible, 1.47 and 1.44 respectively, and 1.46 considering all cases.

**TABLE 7 CALIBRATION RESULTS, SINGLE SAFETY FACTOR, TARGET PROBABILITY OF FAILURE  $10^{-4}$**

Type of analysis	Format 1, Single Safety Factor		
	All	Haltenbanken	Gulf of Mexico
Quasi static	1.66	1.60	1.67
Dynamic	1.46	1.47	1.44

**Two safety factors**

Similar results with design format 2 are included in Table 8.

**TABLE 8 CALIBRATION RESULTS, TWO SAFETY FACTORS, TARGET PROBABILITY OF FAILURE  $10^{-4}$**

Type of analysis	Format 2, Two Safety Factors			
	$\gamma_{ZM}$	$\gamma_{ZD}$		
		All	Haltenbanken	Gulf of Mexico
Dynamic analysis	1.0	1.55	1.51	1.60
Dynamic analysis	1.1	1.52	1.51	1.53
Dynamic analysis (*)	1.06	1.53		

(\*) Both safety factors are optimised simultaneously

The purpose of this format is to improve the consistency in the reliability for all water depths. The format has therefore not been applied to the quasi-static analysis because the effect of line dynamics is important and must be accounted for in deep water. The format includes two safety factors, one on the static tension component at mean offset  $\gamma_{ZM}$ , and one on the dynamic low-frequency and wave-frequency tension component  $\gamma_{ZD}$ . Based on Fig. 5, it seems reasonable to choose the safety factor for the tension at mean offset near unity. The safety factor for the time varying tension has been calibrated with alternative safety factors of 1.0 and 1.1 for the tension at mean offset. The calibration has also been performed optimising both safety factors simultaneously.

For  $\gamma_{ZM}$  equal to 1.0, the Gulf of Mexico requires a safety factor for the dynamic tension slightly higher than at Haltenbanken, i.e. 1.60 versus 1.51. This difference practically vanishes when  $\gamma_{ZM}$  is increased to 1.1, with the corresponding  $\gamma_{ZD}$  of 1.51 and 1.53 for Haltenbanken and the Gulf of Mexico respectively. When both safety factors are optimised simultaneously, the obtained values are 1.06 and 1.53 for  $\gamma_{ZM}$  and  $\gamma_{ZD}$  respectively.

**Resulting Reliability Level**

Fig. 7 shows the reliability obtained for the various cases by using the calibrated safety factors based on (a) single safety factor of 1.66, quasi-static analysis excluding deep water cases, (b) single safety factor of 1.46, dynamic analysis, and (c) safety factor of 1.1 for the tension at mean offset and 1.52 for dynamic low- and wave-frequency tension. Generally one can say that a quasi-static analysis becomes less conservative with increasing water depth. Dynamic analysis, which is generally required for deep water, shows improved reliability with increasing water depth.

It may be seen in Fig. 7 that the format with two safety factors leads to a more uniform safety level closer to target than a single safety factor format.

The governing cases in terms of lowest reliability are the ship in 70 m at Haltenbanken with the Jonswap spectrum and the semisubmersible in 350 m water depth in the Gulf of Mexico. The effect of two safety factors is a minor improvement in the reliability for the first of these cases, whereas the reliability becomes more critical for the semisubmersible in 350 m. This is because the tension at mean offset is a significant portion of the total tension for this case.

Fig. 7 also shows that the overall reliability level for the semisubmersible is higher than for the ship, but this is less pronounced when two safety factors are used. The effect of two safety factors seems to work as intended, i.e. the overdesign in deep water is significantly reduced. (The lowest probabilities of failure in Fig. 7, say below  $10^{-7}$ , may be inaccurate due to extrapolation.)

**Required Line Strength**

The design tension (=required mean line strength) has been calculated for the various cases by using the calibrated safety factors based on (a) single safety factor of 1.66, quasi-static analysis excluding deep water cases, (b) single safety factor of 1.46, dynamic analysis, and (c) safety factor of 1.1 for the tension at mean offset and 1.52 for dynamic low- and wave-frequency tension. The results have been normalised to the design point tension obtained in the reliability analysis at  $10^{-4}$  annual probability of failure, and are included in Fig. 8. It may be seen that quasi-static analysis generally gives a conservative design in shallow water. In the case of dynamic analysis, overdesign of around 40% using a single safety factor of 1.46 is typically reduced to almost the half when two safety factors are applied. With two safety factors the required line strengths of all the 20 cases considered are between 95 % and 124 % of the target line strength from the reliability analysis.

**CONCLUSION**

A turret moored storage tanker in 70 m, 350 m and 2000 m water depth, and a semisubmersible in 70 m, 350 m and 1000 m water depth have been subjected to structural reliability analysis with respect to an ultimate limit state for single mooring line failure.

Three formats for the design equation have been considered:

- (a) quasi-static mooring system analysis with a single safety factor
- (b) dynamic mooring line analysis with a single safety factor
- (c) dynamic mooring line analysis with two safety factors

The environmental actions to be applied in the line response computations should be based on wind and wave conditions with a 100-year return period, applied together with current with a 10-year return period. The wave conditions should include a set of combinations of significant wave height and peak wave period along the 100-year determined using the inverse FORM technique. Collinear and non-collinear environmental actions should be considered. The characteristic tension components should be computed according to the recipe specified by API RP 2SK (1995), with the modifications given above.

The characteristic line strength should be computed from the mean value and coefficient of variation of the chain link or wire rope components, as specified by equation (6). If this information is not available, then the characteristic line strength may be taken as the minimum breaking strength multiplied by a reduction factor of 0.95. Other factors may be needed for old lines.

With design format (c), and the other recommendations above, the following partial safety factors are recommended:

- 1.10 applied to the characteristic static tension, and
- 1.52 applied to the characteristic dynamic tension.

If a quasi-static analysis is used in design format (a), then a partial safety factor of 1.66 should be applied to the characteristic tension.

The use of two safety factors has been shown to improve the consistency between designs in shallow and deep water. No additional safety factor needs to be applied to the characteristic line strength.

In discussions after this calibration, it has become apparent that the proposed design rule will be very inflexible, and that it will be difficult to incorporate the results of more refined analyses into the design process. The reliability analyses have shown that the probability of failure in the ULS is dominated by uncertainty in the line tensions. Many designers will naturally be interested in more detailed analyses of the tensions and want to take account of these results in the design. To alleviate this problem, we intend to make allowance for the use of alternative recipes or methods in the computation of the characteristic loads. A "method factor" will be associated with each recipe, and included in the design equation. One additional recipe will be considered initially, but the number of recipes may be extended as the need arises.

If a designer is not satisfied by the freedom and accuracy provided by the design rule outlined above, then direct application of reliability analysis to the design should be an option.

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FIGURES

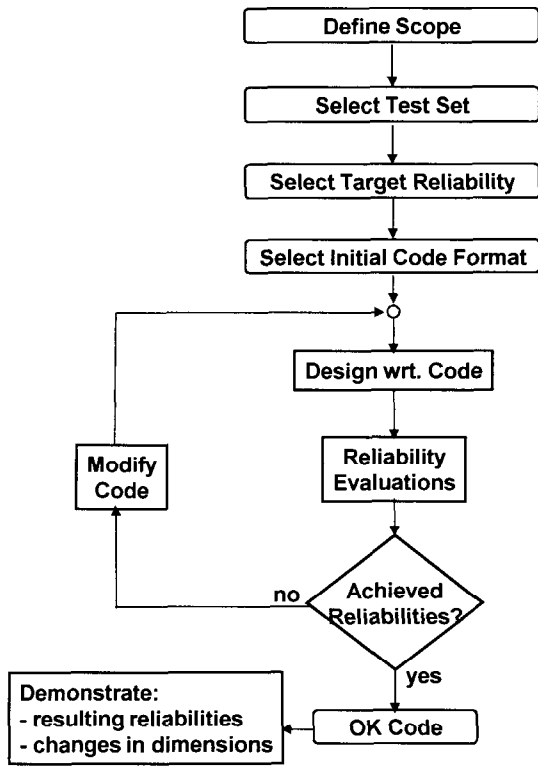


FIG.1 OVERVIEW OF THE CALIBRATION PROCESS.

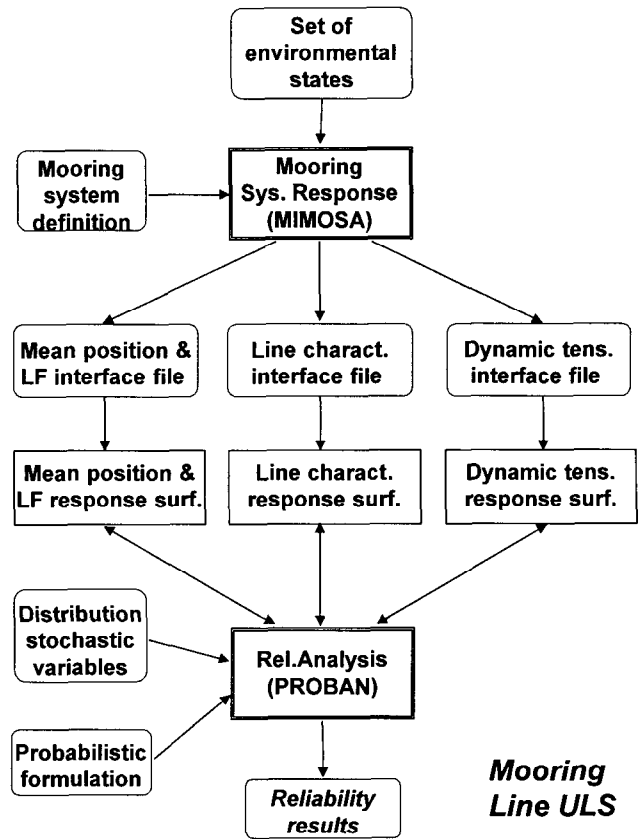


FIG. 2 OVERVIEW OF THE DATA FLOW IN THE ULS CALCULATION

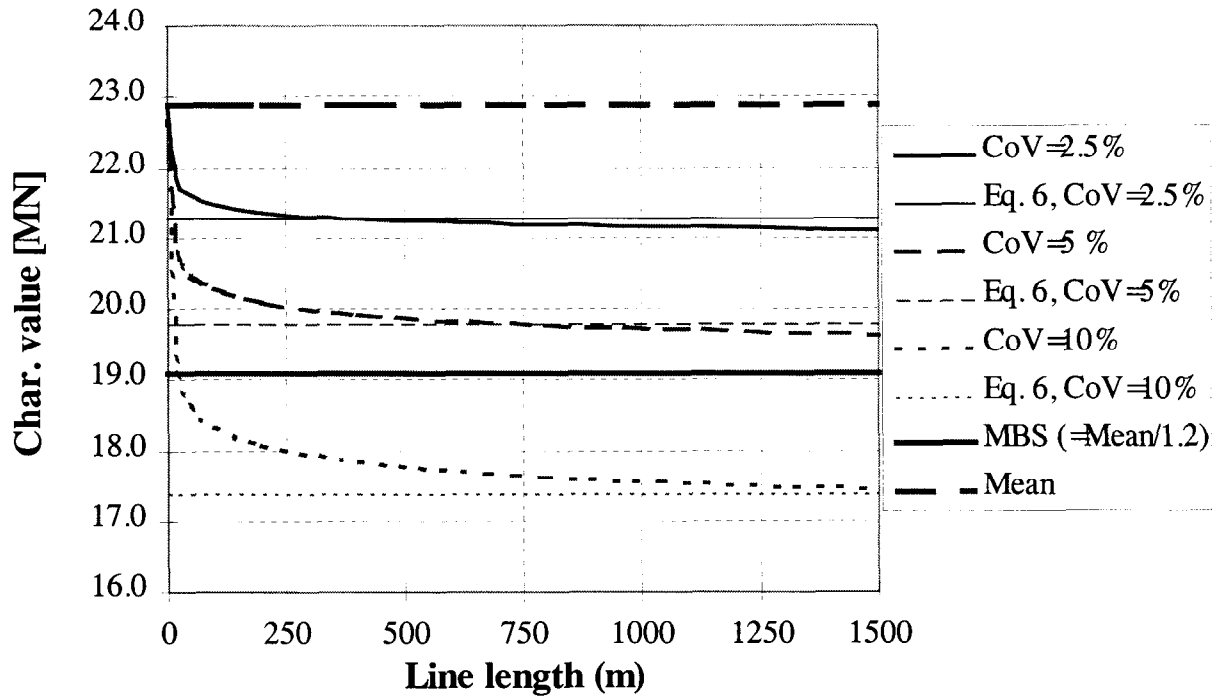


FIG. 3 THE EFFECT OF DIFFERENT DEFINITIONS FOR THE CHARACTERISTIC LINE STRENGTH: (A) THE MEDIAN STRENGTH VARIES WITH LENGTH, (B) SIMPLIFIED FORM IN EQUATION (6) IS INVARIANT WITH LENGTH.

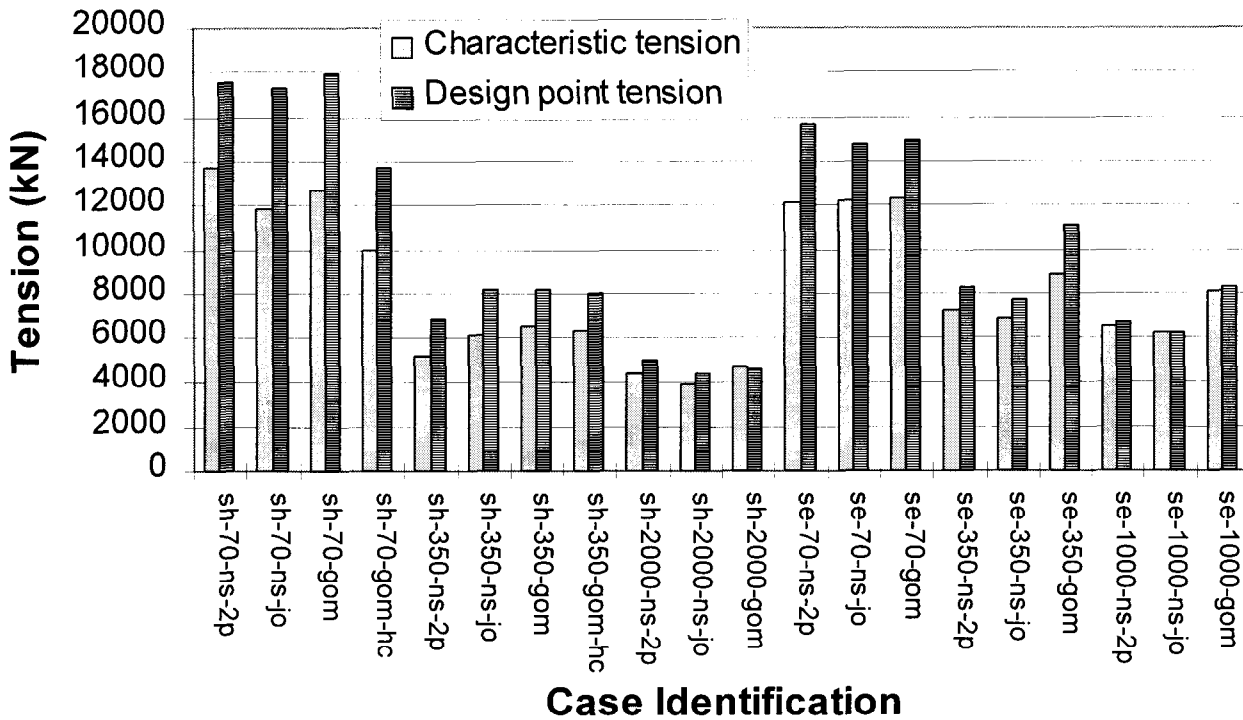


FIG. 4 CHARACTERISTIC AND DESIGN POINT TOTAL TENSION AT  $10^{-4}$  PROBABILITY OF FAILURE

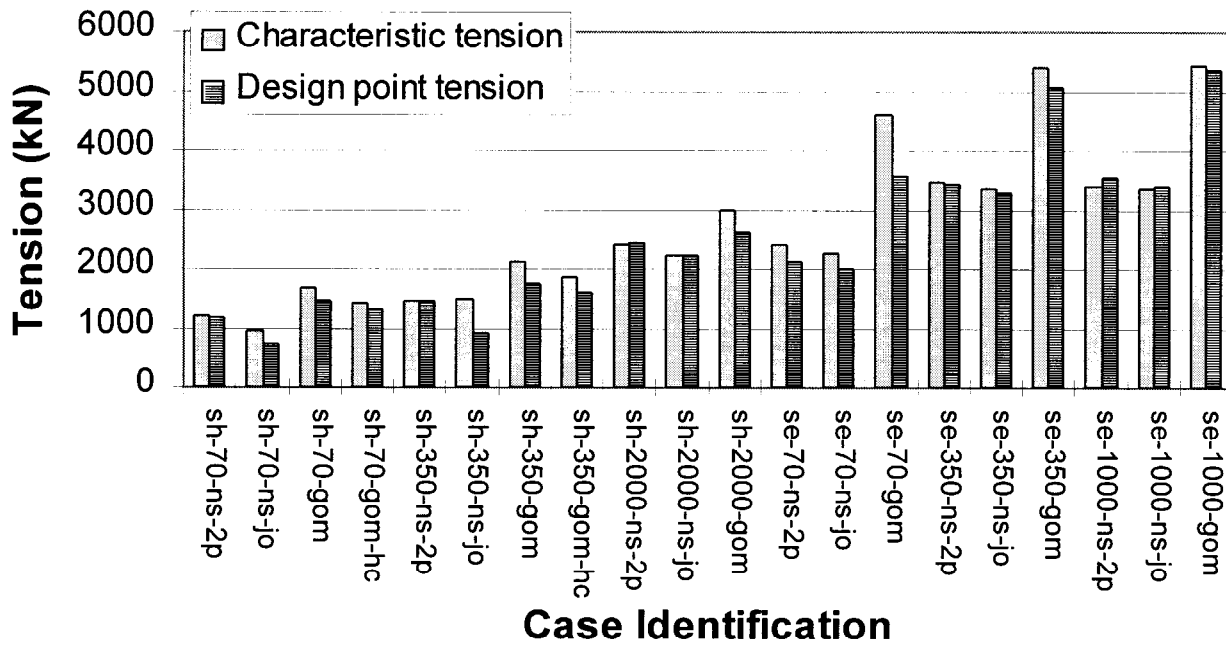


FIG. 5 TENSION AT MEAN OFFSET. COMPARISON OF CHARACTERISTIC AND DESIGN POINT TENSION COMPONENTS AT  $10^{-4}$  ANNUAL PROBABILITY OF FAILURE.

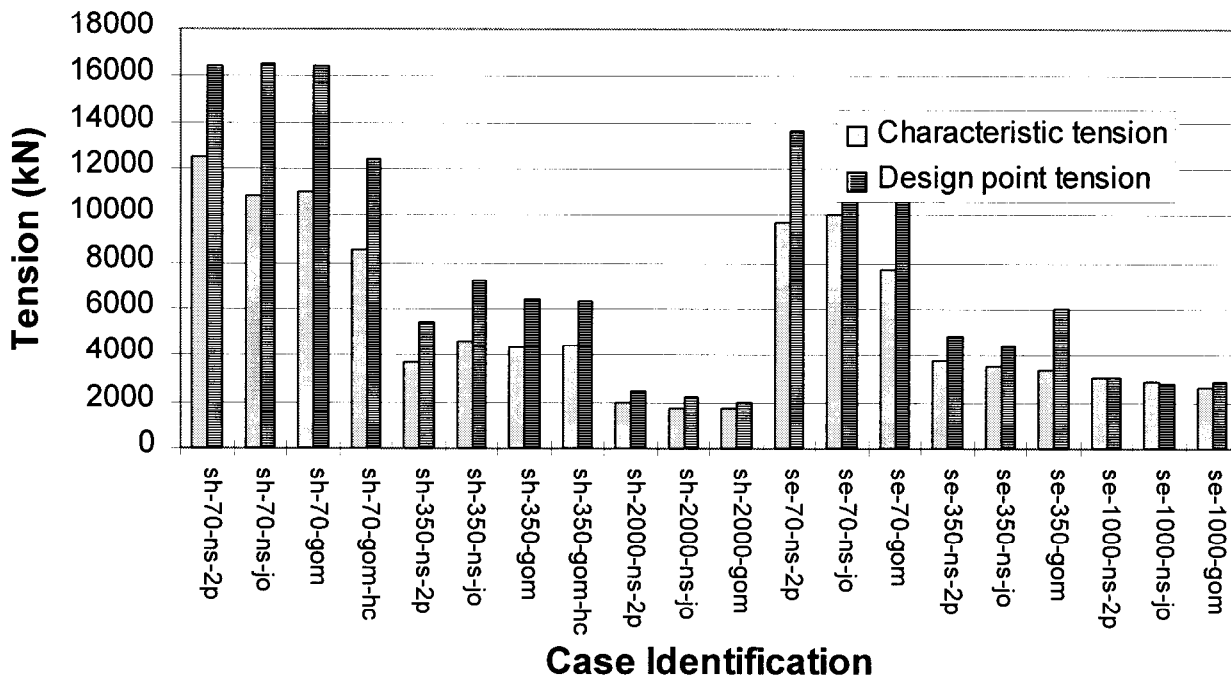


FIG. 6 DYNAMIC, I.E. OSCILLATING LOW-FREQUENCY PLUS WAVE FREQUENCY TENSION. COMPARISON OF CHARACTERISTIC AND DESIGN POINT TENSION COMPONENTS AT  $10^{-4}$  ANNUAL PROBABILITY OF FAILURE.

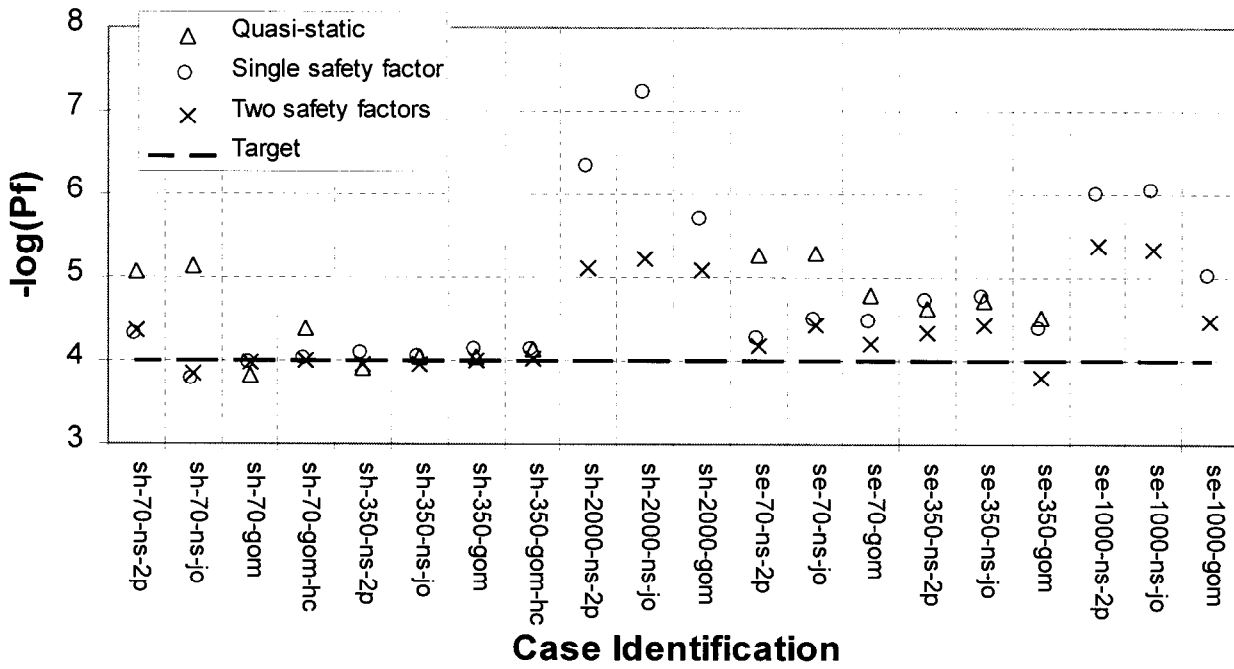


FIG. 7 RESULTING RELIABILITY LEVEL,  $-\log(P_f)$ , AFTER CALIBRATION. QUASI-STATIC ANALYSIS WITH SINGLE SAFETY FACTOR, AND DYNAMIC ANALYSIS WITH SINGLE AND TWO SAFETY FACTOR FORMAT.

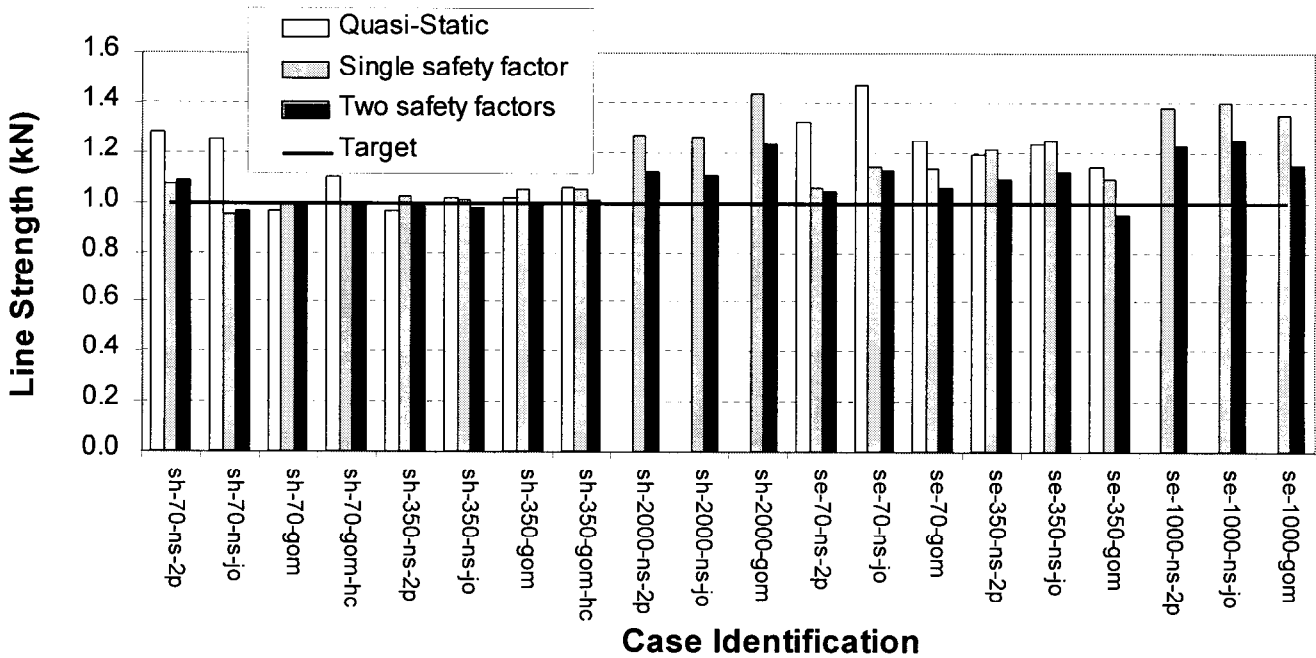


FIGURE 8. REQUIRED LINE STRENGTH AFTER CALIBRATION. QUASI-STATIC ANALYSIS WITH SINGLE SAFETY FACTOR, AND DYNAMIC ANALYSIS WITH SINGLE AND TWO SAFETY FACTOR FORMAT. THE RESULTS ARE NORMALISED TO THE DESIGN POINT VALUES AT  $10^{-4}$  ANNUAL PROBABILITY OF FAILURE.