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CALIBRATION OF A PROGRESSIVE COLLAPSE LIMIT STATE FOR MOORING LINES

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ABSTRACT

A structural reliability analysis is formulated for a progressive collapse limit state of offshore mooring systems. The failure of two adjacent mooring lines is considered in some detail, while incorporating the empirical frequency of line failures in the modelling of the initial line failure. Reliability results are presented for the mooring system of a semisubmersible platform, and a turret-positioned ship. A simplified design rule for the progressive collapse limit state is discussed and the reliability results are utilised in a preliminary calibration of the design rule. Additional test cases are required to finalise the calibration.

INTRODUCTION

Three criteria should be considered in the structural design of mooring lines for floating offshore structures. Within a structural reliability format it is convenient to formulate these criteria as: (a) an ultimate limit state (ULS) to ensure that each mooring line is strong enough to withstand the extreme loads it is subjected to, (b) a progressive collapse limit state (PLS) to ensure that the mooring system can withstand the failure of one mooring line due to other causes, and (c) a fatigue limit state (FLS) to ensure that each mooring line has adequate capacity against fatigue. This paper deals with the PLS, or the accidental limit state, while companion papers deal with the other two limit states (Hørte et al. 1998, and Lie et al., 1998). The results are intended for use in the revision of the Posmoor rules for mooring line design (Sogstad, 1998). There is also usually a serviceability requirement in the design of mooring lines, to ensure that the motion of the platform does not exceed limits imposed by attached risers or adjacent structures. This is obviously essential for a satisfactory design, but it is convenient to separate the serviceability requirement from the requirements placed on the strength of the mooring lines. The serviceability is usually adjusted by means of the line pretension, elasticity, weight, or number of lines. After changing any of these parameters it is necessary to check that all limit states are still satisfied.

The objective of this work is to calibrate a simplified design method for the PLS, against a detailed structural reliability analysis of the PLS, such that a chosen target reliability level is achieved when the design method is applied. Only a few test cases for offshore mooring systems have been considered at the time of writing, and more test cases will be included before the calibration is finalised.

Ahilan et al. (1996, 1997) have included consideration of first and second mooring line failures together with riser analysis, in applications to Foinaven and Schiehallion floating production systems. There is some similarity in their approach, but the present paper is considerably more detailed in the formulation and discussion of the reliability analysis of the mooring lines.

CALIBRATION

Much of the content of this paper deals focuses on a detailed reliability analysis of a mooring system PLS. The objective of this analysis is to calibrate a simplified design method for a mooring system PLS. This paper leans heavily on the companion paper for the ULS by Hørte et al. (1998), which also includes some discussion of the calibration problem, and the calibration methods used in both analyses.

A set of test structures has been selected to span the scope of the calibration. These include a turret-positioned ship and a semisubmersible, with various mooring systems for water depths of 70 m, 350 m, 1000 m and 2000 m. Only the 350 m water depth is included in the present paper. Environmental conditions for the Norwegian continental shelf are considered herein, but the Gulf of Mexico will also be included.

SYSTEM RESPONSE

Time Intervals

The mooring system response to an initial line failure can be split into three time intervals: (I) transient response immediately following the initial failure, (II) short-term, stationary response in the same environmental state as the initial failure, but making allowance for the missing line, and (III) long-term response in various environmental states until the missing line is repaired or replaced.

Transient Response in Interval I

A detailed analysis of the transient response in interval I has been carried out by Mathisen et al. (1996). Both analytical results and time simulation results were obtained, and it was found that the offset of the mooring system during the transient is unlikely to exceed the offset during the stationary conditions following the transient. Moreover, if the offset in interval I should exceed the offset in interval II, then it will not do so by a large amount. Hence, further analysis of interval I can be avoided, provided that interval II is covered.

Some examples from these results for transient response are given in Table 1. The main result in each line of Table 1 is the relative offset; i.e. the ratio of the offset during the transient response to the offset in the subsequent stationary response. This is given for time history simulations in irregular waves using the

Type	Damp. ratio	Force loss ratio	Fail. Inst.	RELATIVE OFFSET Simu-lation	Ana-lytical
Ship	0.23	0.3	2	0.98	0.94
Ship	"	0.3	1	0.87	0.92
Ship	"	0.44	2	0.83	0.95
Ship	"	0.44	1	0.58	0.91
Ship	0.13	0.30	1	0.73	0.86
Ship	"	0.36	1	0.92	0.86
Ship	"	0.44	1	0.65	0.85
Ship	0.08	0.88	2	0.93	0.93
Ship	"	0.88	1	0.61	0.78
Ship	"	1.0	2	0.95	0.93
Ship	"	1.0	1	0.74	0.76
Semi	0.27	0.09	2	0.92	0.96
Semi	"	0.10	1	0.88	0.96
Semi	0.28	0.27	2	0.95	0.96
Semi	"	0.28	1	0.92	0.95

TABLE 1 RATIO OF TRANSIENT OFFSET AT INITIAL LINE FAILURE TO SUBSEQUENT STATIONARY OFFSET IN THE SAME ENVIRONMENTAL STATE

SIMO program (Marintek, 1993), and for analytical response to simple harmonic excitation. Two possible line failure instants are included: (1) failure at maximum offset, and (2) failure at maximum velocity. A range of system parameters is covered: for the critical damping ratio, and for ratio of lost restoring force to the mean environmental force. A single, realistic value is applied for the ratio of the dynamic environmental force to the static environmental force. Separate stiffness ratios are applied for the ship and semisubmersible, to specify the ratio of linearised mooring system stiffness before failure to the stiffness after failure. The offset ratio is less than unity in all the cases in Table 1. Offset ratios up to 1.12 were obtained from the analytical model for some parameter values, when the initial line failure was assumed to occur at minimum offset. However, minimum offset is the part of the offset motion cycle that causes the least line tension and seems physically implausible as the failure instant.

The above conclusion on transient response applies in the presence of oscillatory environmental loads, but not in calm water. Calm water conditions will not normally pose a comparable risk of a second line failure, but should be investigated in more detail if unusually high line tensions should be deliberately applied for reasons which are unforeseen at present.

Response Calculation

The response of the mooring system in intervals II and III is calculated using the MIMOSA program (Lie, 1990). The parameters of the short-term response distributions are collected on an interface file for a wide range of environmental conditions. A response surface technique (Mathisen, 1993) is applied to make the short-term response available to the reliability analysis. This response analysis is essentially the same as applied in the ULS, described by Hørte et al. (1998), and by Larsen and Mathisen (1996).

In the reliability analysis of the PLS, it is insufficient to simply consider an intact mooring system and the tension in a single line, as is done in the ULS. First the tension in the line which initially fails must be considered, with the system intact. This is needed to analyse the conditions under which that line fails. Subsequently, the tension in an adjacent line must be considered, taking account of the missing line in the mooring system response. The same calculation procedures can be used to analyse both cases, but now two sets of interface files have to be handled. It would be arduous to extend this detailed analysis to consider the possibility of failure of a third line.

RELIABILITY ANALYSIS

This PLS analysis is limited to consideration of the failure of two adjacent mooring lines. This is logically the first step after analysis of a single failure in the ULS, and considerably simpler than complete system failure. However, the response characteristics of spread mooring systems are such that two line failures in unfavourable weather are quite likely to be followed by additional line failures, and lead to system failure. Hence, the reliability against two line failures is likely to be a useful indication of the mooring system reliability. This indication will be most accurate if the most heavily loaded pair of lines is analysed in a system with identical lines. The indication will be somewhat unconservative if all the lines are individually designed at the same design limit or reliability.

Practical experience with mooring lines shows that line failures most frequently occur because something "exceptional" has happened (ref. Andreassen et al., 1993, Tein, 1995, Miller, 1990, Stevens, 1997), such as: errors in production that are not detected by quality control procedures, damage inflicted in transportation, installation, or through faulty operation, fatigue damage, excessive corrosion or mechanical wear, gross errors in design and verification, climatic change, or other unknown causes. We may generalise by saying that the mooring line strength or tension is "exceptional" through not being drawn from the distribution functions that are normally applied in design. To provide a rational reliability analysis we have to quantify this "exceptional" quality in our reliability model, in a reasonably accurate way. Some empirical data exists to quantify the frequency of mooring line failures, and this frequency is subsequently applied in the present model.

Line Combinations

A line failure may arise if the line is "exceptionally" weak or the line tension is "exceptionally" high. Practical experience indicates that the line failures occur most frequently due to reduced strength, and this is the category that will be considered explicitly here; i.e. each line is assumed to be either "O for ordinary" or "W for weak." When considering two adjacent lines, three possible combinations have to be considered: OO, WO, and WW. The probability of failure of the two lines includes the probability of failure of each combination, and the probability that that combination arises

$$P_f = P_{f|oo} \cdot P[OO] + P_{f|wo} \cdot P[WO] + P_{f|ww} \cdot P[WW] \qquad (1)$$

where $P[.]$ indicates a probability, $|$ indicates conditioning, and the use of W and O should be obvious. The probabilities of occurrence of "weak" and "ordinary" lines are taken to be time-independent. This is not completely accurate, since the transition from "ordinary" to "weak" must occur at some point in time, but it is relatively accurate compared to the obvious time dependency involved in the load processes. The present analysis is primarily aimed at the WO combination, because practical experience indicates that this is where design effort is needed to ensure acceptable redundancy in mooring systems.

Failure of a "Weak" and an "Ordinary" Line

A limit state function for failure of one line may be written as

$$g = s - z \qquad (2)$$

where s is the line strength and z is the applied tension. The limit state function g takes positive values when the line is intact and negative values if the line fails. Now let us consider a "weak" and an "ordinary" line, with failure events denoted C_w and D_o, respectively, and with corresponding indices applied to their respective limit state functions. Then the conditional probability that both lines fail is given by

$$P_{f|wo} = P\left[C_w \bigcap D_o\right] = P\left[G_C < 0 \bigcap G_D < 0\right] \quad (3)$$

where $\bigcap$ indicates an intersection of two events, and capital letters are used to indicate stochastic variables when appropriate. The event for first line failure C is computed for an intact mooring system, while the event for second line failure D is computed with the adjacent line missing. The stochastic strength of the "exceptionally weak" mooring line is denoted by S_w. The "weak" line may be expected to fail first, such that the tension Z_C may be computed for an intact mooring system, as in the ULS analysis. The strength of the second line S_o is assumed to be ordinary, and drawn from the same strength distribution as applied in the ULS analysis, while the tension Z_D in the second line is calculated with the first line missing, otherwise using the same analysis procedures as applied in the ULS analysis.

Interval III

The initial failure event C_w has already occurred before interval III starts. Thus, the intersection event in equation (3) is simplified, and it is only necessary to compute the probability of event D_o. The analysis of interval III can then be handled similarly to the ULS analysis. Adjustment of line pretensions may be taken into account in interval III, but would not be considered for interval II. The time duration between the initial failure and the subsequent repair is taken into account in the extreme value distribution of the applied tension Z_D.

Interval II

Interval II requires detailed analysis of the intersection event in equation (3), and seems likely to be the more critical case. In interval II, both line failures occur in the same environmental state. Hence the occurrence of the first failure implicitly affects the probability of the second failure. If the "exceptional" line is very weak, then it will fail in relatively mild environmental conditions, and the second line is likely to survive. If the "exceptional" line is relatively strong, then it will fail in more severe environmental conditions, and the second failure is more likely. Thus, it seems essential to carefully model the environmental conditions under which both failures occur. An arbitrary choice of these conditions must lead to an arbitrary probability of failure, with little meaning. Hence, the formulation may usefully proceed by considering events conditional on a specific environmental state

$$P_{f|wo,\Psi}(\psi) = P\left[(C_w|\Psi) \bigcap (D_o|\Psi)\right] \quad (4)$$

where Ψ denotes the stochastic vector defining the environmental conditions, including significant wave height, peak wave period, mean wind speed, current speed, and the directions of these environmental effects with respect to the moored platform. The capacities in the limit state functions are independent of the environmental conditions, but the tensions are dependent on the environmental conditions. Hence, quite extensive information about the tension has to be available in the reliability analysis, to be able to compute both the tension distributions in all relevant environmental states. However, the intersection event in equation (3) is not exactly what we are seeking. As it stands, this expression will include all environmental states that lead to failure of both lines. We wish to exclude those environmental states that are more severe than necessary to fail the weaker line, because we know that this line will already have failed in a preceding, milder state. For this purpose, we include a second intersection, with a third event, while replacing the event symbols with the underlying stochastic variables, as

$$P_{f|wo,\Psi}(\psi) = P\left[(Z_{C1}(\psi) > S_w) \bigcap (Z_D(\psi) > S_o) \bigcap (Z_{C2}(\psi) < S_w)\right] \quad (5)$$

where we have introduced two tension variables related to the line which fails first, Z_{C1} and Z_{C2}. The middle event states that the tension exceeds the strength of the second, "ordinary" line. The first event states that some tension Z_{C1} exceeds the strength of the "weak" line, while the last event states that some other tension Z_{C2} is less than the strength of the "weak" line. In our analysis, we have chosen to model these two tensions in the "weak" line as two independent realisations of the short-term, extreme value distribution of the tension. This seems to work well. An equality event with the tension in the "weak" line equal to the strength of the "weak" line C might seem preferable, intuitively. However, this would meet formal objections, because there is zero probability that a continuous stochastic variable is equal to a point value. Equation (5) may be considered as an approximation for a more detailed formulation that would take account of the time variation of the environmental states – which exceeds the level of complexity we are prepared to consider here.

The short-term, extreme value distribution of the tensions in both lines is modelled as in the ULS analysis, with all lines intact in the first case, and the "weak" line missing in the second case. The angle between the two lines, and the relative heading angles with respect to the environmental effects are also taken into account. The intersection event expressed by equation (5) is computed using a first order reliability method (FORM) or a second order reliability method (SORM) with the PROBAN program (DNV, 1996). Since heading angles are strongly involved in the system response, with cosine-like behaviour, this tends to introduce non-linearity in the limit state function (when transformed to the space of independent standard normal variables), and require use of SORM rather than FORM. This computation should be done conditionally with respect to the time independent variables, such as the two line strengths, and any model uncertainties that are involved, but have not been shown explicitly here.

The theorem of total probability is applied to obtain the marginal probability of failure in a single, random, environmental state

$$P_{f|wo,S_w,S_o}(s_w, s_o; \tau) = \int P_{f|wo,S_w,S_o,\Psi}(s_w, s_o, \psi) f_\Psi(\psi) d\psi \quad (6)$$

where τ is the duration of an environmental state, $f_\Psi(\psi)$ is the joint probability density function of the environmental variables, and the conditioning with respect to the time-independent strength variables is now shown explicitly. The reliability integral in equation (6) and the intersection probability in equation (5) are computed in a single FORM or SORM computation. Allowance is next made for the number of environmental states encountered in a long-term time duration λ, while assuming independence between states

$$P_{f|wo,S_w,S_o}(s_w, s_o; \lambda) = 1 - \left[1 - P_{f|wo,S_w,S_o}(s_w, s_o; \tau)\right]^{\lambda/\tau} \quad (7)$$

The theorem of total probability can then be applied to integrate out the conditioning on the time-independent variables

$$P_{f|wo}(\lambda) = \int P_{f|wo,S_w,S_o}(s_w, s_o; \lambda)\, f_{S_w}(s_w) f_{S_o}(s_o)\, ds_w\, ds_o \quad (8)$$

where $f_{S_w}(s_w)$ and $f_{S_o}(s_o)$ are the probability density functions for the strengths of the respective lines. The probability integral in equation (8) is computed by an additional FORM or SORM layer outside the inner layer for equations (5) and (6).

Finally, we have to make allowance for the probability of the "weak" and "ordinary" combination of two lines (WO). Let us assume that there is a probability q that any line is "exceptionally weak." Then we may apply the binomial distribution to compute the relevant probabilities in equation (1)

$$\begin{aligned} P[OO] &= (1-q)^2 \approx 1 \\ P[WO] &= 2q(1-q) \approx 2q \\ P[WW] &= q^2 \end{aligned} \quad (9)$$

where the approximations are valid when q is small. In principle, all three possibilities from equation (1) should be included when evaluating the probability that two lines fail. In practice, the case with two "ordinary" lines is expected to contribute relatively little to the probability of failure. The case with two "weak" lines has not been computed, and is not very useful in design, because there is no quantitative basis to link this case to change in the strength of "ordinary" lines. Hence, we focus on the case with a single "weak" line, and include an asterisk in the notation, as a reminder of this simplification

$$P_{f^*}(\lambda) = P_{f|wo}(\lambda) \cdot 2q(1-q) \quad (10)$$

There is little to be gained by reducing this probability much below q^2, because the occurrence of two "weak" lines will then dominate the probability of failure. However, the value of q may fall as procedures related to mooring lines are improved.

Model for "Weak" Lines

The formulation above is based on knowledge of the probability of occurrence of a "weak" line q and the distribution function for the strength of "weak" lines S_w. Reasonably accurate empirical data is available for the annual probability of failure, but for little more than this. Andreassen et al. (1993) give some information on the line tension at failure, which was initially used to model the line strength at failure, but subsequently abandoned as being too unreliable. Instead a plausible family of distribution functions have been assumed for the line strength of the "weak" lines. This is based on the beta distribution, with probability density given by

$$f(x) = \frac{(x-a)^{r-1}\,(b-s)^{s-1}}{(b-a)^{s+r-1}\,B(r,s)} \quad (11)$$

where a is the lower bound, b is the upper bound, r and s are parameters, and the beta function is defined by

$$B(r,s) = \int_0^1 t^{r-1}\,(1-t)^{s-1}\, dt \quad (12)$$

Initial line failures during installation and tensioning are discounted in interval II, and the lower bound for the "weak" line strength is set equal to the pretension. The upper bound for the "weak" line strength is set equal to the mean strength of ordinary lines. Parameter s is set equal to unity, to provide a J-shaped density function, implying increasing probability density as the strength approaches the upper bound. This requirement was not set initially, and its omission appeared to cause inconsistent behaviour of the reliability results in a sensitivity check. The last parameter r can be adjusted to calibrate the strength distribution against the empirical frequency of line failure. Fig.1 shows the beta density function for a few different values of the r parameter, which should exceed unity to provide the required J-shape.

The annual probability of failure of a random mooring line =0.0074, according to Andreassen et al. (1993), based on empirical data for chain produced after 1985, and excluding installation and tensioning operations. Assuming the majority of the empirical data is related to "weak" lines, this probability is approximately equal to the probability of a "weak" line q multiplied by the annual probability that a "weak" line fails (event C_w above). We have assumed that a "weak" line is fairly likely to fail in the course of a year and set q=0.01. Then the annual probability of failure of a "weak" line becomes 0.0074/q=0.74, and the parameter r can be adjusted so that the reliability analysis of a "weak" line yields this probability.

DESIGN ANALYSIS

The initial format of the design equation is based on the same format as developed for the ULS, for convenience (Hørte et al., 1998). This design equation is written as

$$s_C - z_{CM} \cdot \gamma_{ZM} - z_{CD} \cdot \gamma_{ZD} \geq 0 \quad (13)$$

where s_C is the characteristic line strength, z_{CM} is the characteristic line tension at the mean offset, z_{CD} is the characteristic increase in dynamic line tension due to both low-frequency and wave-frequency platform motions, and γ_{ZM} and γ_{ZD} are the corresponding partial safety factors. The detailed recipes for the characteristic values are specified for the ULS (ibid). The interpretation of the components is modified somewhat for the PLS. The definition of the characteristic strength is unchanged. The two tension components are computed for a line adjacent to a line which is assumed to have failed, and taking into account the absence of the failed line. The partial safety factors are to be calibrated from the present analysis. A contour of characteristic environmental conditions, mainly based on a 100-year return period is specified for the ULS (ibid), and the same environmental conditions are initially specified for the PLS, for convenience.

TEST CASES

Only two test cases are included in the present paper, while additional cases will be included in the final calibration. These two cases comprise a turret-moored ship and a semisubmersible, both in 350 m water depth. The ship has 8 mooring lines in a rotationally symmetric pattern, while the semisubmersible has 12 lines spread in 4 groups of 3 lines. The mooring lines include chain and steel wire rope segments. Further details are given in the paper on the ULS (Hørte et al., 1998).

The details of the distribution functions that are applied in the reliability analysis are the same as for the ULS (ibid), with the addition of the weak line, as described above.

RESULTS

Reliability Results

There is some practical difficulty in reliability calculations which are strongly affected by heading angle and cosine-like functions, as indicated above. Fig.2 and Fig.3 are intended to illustrate these effects. The distribution of the line tension is shown in these two figures, excluding all time independent variables, such as model uncertainties. This allows the reliability analysis to be computed in a single layer, using either FORM or SORM. Fig.2 applies to a single line in an intact mooring system and shows exceedence probabilities computed with FORM to be about a factor of 10 higher than those computed with SORM. This is mainly because the first order (FORM) approach cannot capture the heading angle effect properly, in which the tension falls off on both sides of the most unfavourable heading. Fig.2 also shows a steeper slope of the distribution for the semisubmersible, probably because it is less strongly affected by low-frequency motions than the ship. Fig.3 shows a distribution function for a hypothetical tension event; viz. the intersection of the line tension in a line in an intact system and the line tension in an adjacent line with the first line missing, both exceeding the same threshold level. This hypothetical event is closely related to the two line failure event studied in the PLS. Fig.3 shows that the ratio between FORM and SORM probabilities is still about 10 for the semisubmersible, for which the adjacent lines extend in much the same direction. However, there is little difference in the FORM and SORM results for the ship, where there is an angle of 45 degrees between the lines, and the design point is not equally sensitive to heading angle.

The results discussed below include the time-independent variables and require two layers of probability integration. The first order method (FORM) is applied to avoid convergence problems in these computations, and the results are subsequently corrected using the factor between SORM and FORM results obtained above. This seems reasonable when the uncertainties in the load-effects dominate the probabilities, but may introduce some inaccuracy in the results.

Fig.4 shows the annual probability of failure for single lines in an intact mooring system, conditional on the line type; i.e. for an ordinary line and a "weak" line. In this case, r=3.1 in the beta distribution for the "weak" line strength. The target level in the ULS is set to 10^{-4}, and this figure illustrates the ULS requirement for the ordinary line strength of the semisubmersible system at 8.64 MN. Naturally the probability of failure is considerably larger for the weak line. A smaller value of the *r* parameter in the beta distribution for the weak line strength would be required to obtain an annual probability of failure of 0.74.

Two line failures are considered in Fig.5, but still conditional on the line type. The lowermost curve on the figure applies to two lines with "ordinary" strength, and provides a check on the consistency of the "weak" line results – which must be more likely to fail. The uppermost curve shows the highest probability of failure, and applies to a single "ordinary" line, with the adjacent line missing. The two intermediate curves apply to the combined failure of a "weak" and an "ordinary" line. The intersection of 3 events (as in equation (5)) yields slightly lower probability of failure than the intersection of two events (as in equation (4)).

Fig.6 is based on the results in Fig.5, with the probabilities of the line combinations included from equation (9), so these results are not conditional on line type. Now the results appear to be inconsistent, because the curve for two ordinary lines shows a higher probability of failure than the other curves, for a "weak" and an "ordinary" line, and for an "ordinary" line with the adjacent line missing. The curves for two "ordinary" lines and for an "ordinary" line with the adjacent line missing are quite close, and their relative positions do not deserve much comment when the accuracy of the SORM/FORM correction factor is considered. But it does seem unreasonable that the curve for a "weak" and an "ordinary line should show the lowest probability of failure in Fig.6; i.e. that the second term in equation (1) shall contribute less than the first term to the probability of failure in interval II. Two items in the model are critical here: (i) the probability of the line combination $P[WO] \approx 2q \approx 0.02$, and (ii) the modelling of the strength of the "weak" line. We tend to accept the probability of the WO line combination, and the empirical data associated with it. Consequently, we believe that the modelling of the strength of the "weak" lines is at fault. The tuning of the beta distribution for the "weak" line strength against the empirical data is mainly concerned with the low end of the distribution, for very "weak" lines. The two-line failure analysis is mainly concerned with the upper end of the distribution, for relatively strong "weak" lines. Hence, there is some reason to doubt that this tuning is adequate for the present purpose. Therefore, we abandon our attempt to model the strength of the weak lines, and seek some alternative, conservative approximation for the probability of failure of a "weak" and an "ordinary" line. The best available choice seems to be the probability of failure of an ordinary line in the absence of the adjacent line; i.e.

$$P_{f|wo}\left[C_w \bigcap D_o\right] \rightarrow P_{f|wo}\left[D_o\right] \qquad (14)$$

The preceding analysis makes it clear that this must be an annual probability of failure, taking into account all possible environmental states that could cause the combined failure, although interval II is concerned with both line failures in the same environmental state. This is not equivalent to a ULS analysis with one line missing, because the probability of occurrence of the line combination $P[WO]$ is included for the PLS, in equation (1). When this approximation is adopted for interval II, then there is little need to consider interval III in detail, provided that the interval between initial failure and repair is much less than a year.

Fig.7 is based on the approximation in equation (14), and provides the annual probability of failure in interval II for both the ship and semisubmersible test cases, as a function of the mean strength of "ordinary" lines. Only the second term in equation (1) is included, for the "weak" and "ordinary" line combination. This is the primary term that the designer must consider in assessing the redundancy of the mooring system with respect to an "exceptional" event. The first term in equation (1) for the OO-combination should be less than the second term (WO). The third term for the WW-combination should be small. If the WW-term were included, then this would prevent the curves in Fig.7 from showing any further decrease in probability of failure below the level due to the WW-term.

Test Case	ULS	PLS	PLS/ULS
Ship	5.18	6.74	1.30
Semisub.	7.21	9.33	1.30

TABLE 2 CHARACTERISTIC LINE TENSION (MN)

Characteristic Loads

Characteristic loads for the two test cases are given in Table 2, computed as mentioned above. Characteristic loads for the ULS are also included in this table for comparison. The characteristic loads in the PLS are 1.3 times the characteristic loads for the ULS, for these two cases. This ratio is not constant in general, and varies with water depth and between mooring systems.

Partial Safety Factors

The characteristic loads from Table 2 may be combined with the results in Fig.7 to provide the partial safety factors need to achieve a particular reliability level, as shown in Fig.8 and Table 3. The results in this figure are based on a partial safety factor of 1.0 on the mean tension. These results indicate that the 100-year characteristic environment is not unreasonable, provided that a target probability of failure of 10^{-5} or less is applied – because the partial safety factors exceed unity.

Target Level

The target reliability level for the PLS should be chosen to take account of the relatively high likelihood of complete mooring system failure, if the PLS criterion is exceeded; i.e. if two lines fail in interval II. Interval III is not considered any further here, because interval II will normally be more critical. In this case, it is appropriate to consider two consequence classes, where mooring system failure:

(1) is unlikely to lead to unacceptable consequences such as loss of life, collision with an adjacent platform, uncontrolled outflow of oil or gas, capsize or sinking,

(2) may well lead to unacceptable consequences of these types.

The consequence class may be dependent on operational procedures; e.g. a drilling vessel may drill in consequence class (2) under moderate weather conditions, and take measures to isolate the well and transfer to consequence class (1) in severe weather. In such cases, the weather criterion used to separate the consequence classes should be taken into account in the PLS analysis for operation in consequence class (2).

The choice of target level should also take account of the unfavourable environmental conditions expected in interval II. These conditions may leave little time for any action to reduce the consequences of mooring system failure between the initial failure and subsequent line failures.

We suggest that the target level for the probability of failure in consequence class (1) be set to 10^{-5}. This is based on: (a) the discussion above, (b) the target level for the related ULS is set to 10^{-4} and should normally be higher than for the PLS, and (c) comparison with the probability of failure due to two "weak" lines.

The target probability of failure for consequence class (2) would typically be a factor of 10 lower than for class (1), as indicated by the Classification Note on structural reliability analysis (DNV, 1992). This may need some further consideration. Note however, that the impression of the consequences given by Table 3 may be misleading, because there is an additional tension component for the mean tension, and the partial safety factor on this component has been kept constant at unity for both classes.

CONCLUSION

A structural reliability analysis of a progressive limit state for offshore mooring systems has been developed. This limit state is used to check that the mooring system has a reasonable reserve safety if one mooring line should fail due to "exceptional" causes. A mooring system may be expected to easily survive an initial line failure if it occurs in mild weather. The critical question is how the system reacts if the initial failure occurs in a severe environmental state. The analysis tackles this question and models the probability that such a situation should arise.

Various time intervals after the initial failure have been considered. The most critical time interval was found to be in the same environmental state as the initial failure, after transients associated with the initial line failure have decayed. Various attempts were made at detailed stochastic modelling of the strength of the line which fails initially. The possibility that this line has less than "ordinary" strength is a key part of the initial line failure event. These attempts were subsequently abandoned because the "weak" strength could not be adequately quantified from the empirical data of line failures. A somewhat conservative model for the concurrence of initial line failure and second line failure in the same environmental state was adopted instead. The empirical probability of mooring line failure was incorporated in the formulation.

Target level	Factor on mean tension	Factor on dynamic tension	
		Semisub	Ship
10^{-5}	1.0	1.12	1.32
10^{-6}	1.0	1.76	2.12

TABLE 3 PARTIAL SAFETY FACTORS FROM TEST CASES AT TWO TARGET LEVELS

Reliability results have been obtained for two test cases: a ship and a semisubmersible in 350 m water depth. A simplified design method for this PLS has been formulated. The reliability results have been used to compute tentative partial safety factors for the two test cases. Final calibration of the design method is awaiting additional test cases and detailed discussion of the results. The results presented here are quite fresh at the time of writing, and the subsequent discussion may lead to new insights and some changes.

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FIGURES

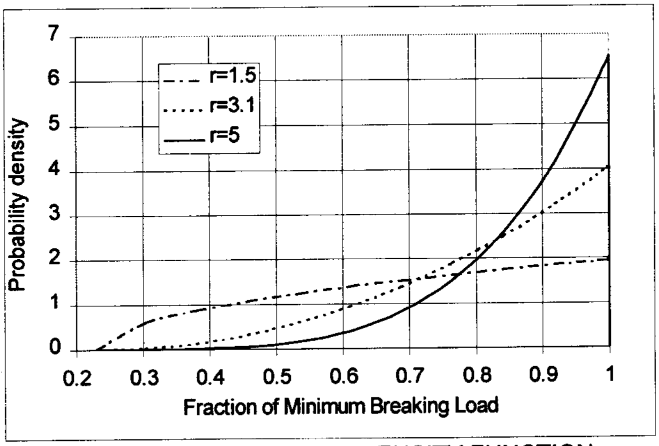

FIG.1 BETA PROBABILITY DENSITY FUNCTION FOR THE STRENGTH OF "WEAK" LINES (LOWER BND.=0.231, UPPER BND=1.0, S=1.0).

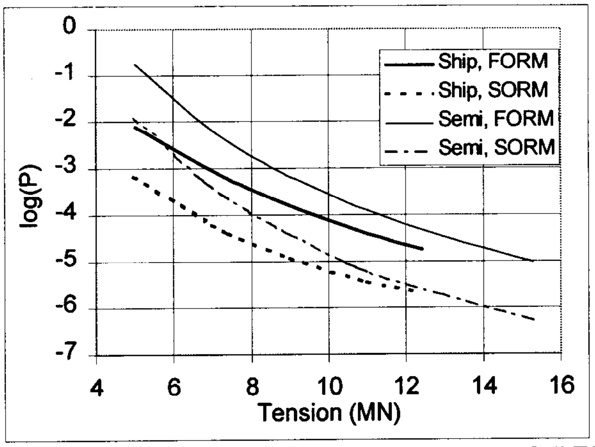

FIG.2 COMPARISON OF FORM AND SORM RESULTS FOR ANNUAL EXTREME VALUE DISTRIBUTION OF TENSION IN A SINGLE LINE OF AN INTACT MOORING SYSTEM (MODEL UNCERTAINTY EXCLUDED).

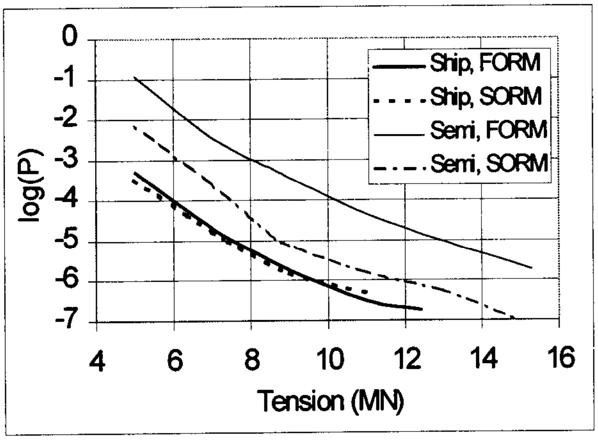

FIG.3 COMPARISON OF FORM AND SORM RESULTS FOR ANNUAL EXTREME VALUE DISTRIBUTION OF TENSION IN TWO ADJACENT LINES (MODEL UNCERTAINTY EXCLUDED).

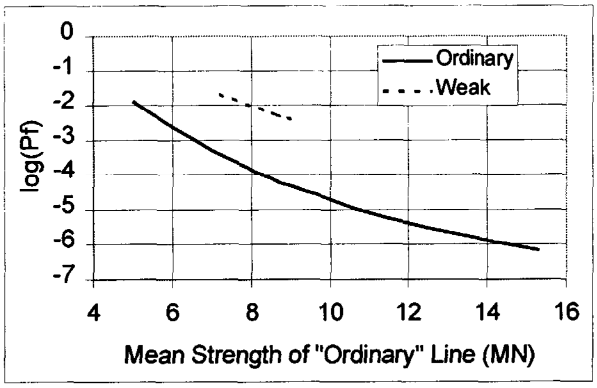

FIG.4 ANNUAL PROBABILITY OF FAILURE OF A SINGLE LINE, CONDITIONAL ON LINE TYPE (SEMISUBMERSIBLE, R=3.1 FOR WEAK LINE)

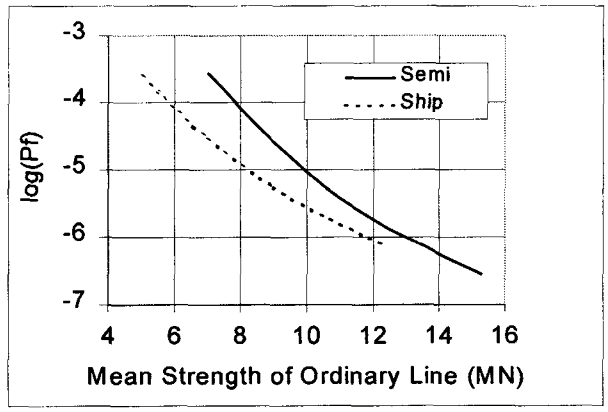

FIG.7 ANNUAL PROBABILITY OF FAILURE OF TWO LINES IN INTERVAL II.

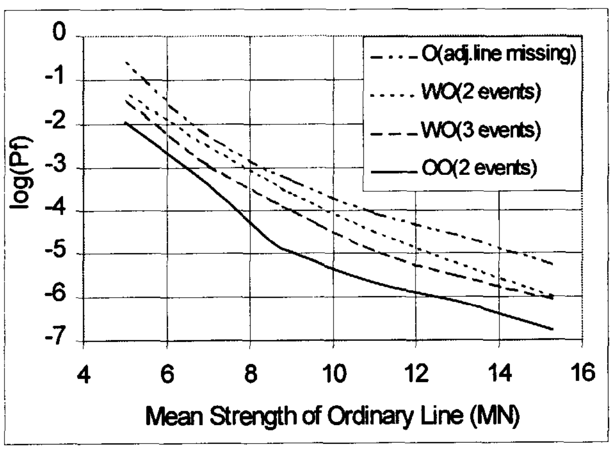

FIG.5 ANNUAL PROBABILITY OF FAILURE OF TWO ADJACENT LINES, CONDITIONAL ON LINE TYPE (SEMISUBMERSIBLE, INT.II).

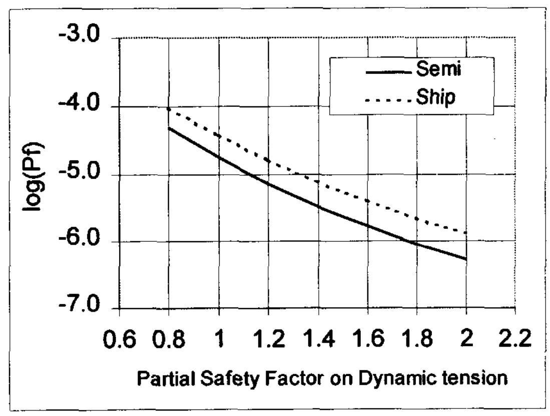

FIG.8 ANNUAL PROBABILITY OF FAILURE FOR TWO LINES IN INTERVAL II AND ASSOCIATED PARTIAL SAFETY FACTOR ON DYNAMIC TENSION.

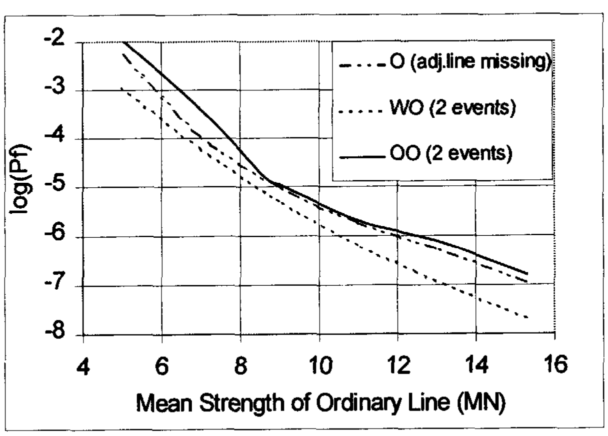

FIG.6 ANNUAL PROBABILITY OF FAILURE OF TWO LINES, INCLUDING PROBABILITY OF LINE TYPE (SEMISUBMERSIBLE, INT.II).